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Weak Field Magnetoresistance in Quasi-One-Dimensional Systems

Yoshitaka NAKAMURA and Hidetoshi FUKUYAMA

Department of Physics, Faculty of Science, University of Tokyo, Tokyo 113

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Theoretical studies are presented on weak localization effects and magnetoresistance in quasi-one-dimensional systems with open Fermi surfaces. Based on the Wigner representation, the magnetoresistance in the region of weak field has been studied for five possible configurations of current and field with respect to the one-dimensional axis. It has been indicated that the anisotropy and its temperature dependences of the magnetoresistance will give information on the degree of one-dimensionality and the phase relaxation time.

KEYWORDS: magnetoresistance, weak localization, quasi-one-dimensional system, organic conductor, Wigner representation

§1. Introduction

Recently, many experiments have reported metallic properties of highly conducting doped polymers (HCDP), *e.g.* polyacetylene doped with iodine,[1)] *p*-phenylenevinylene doped with sulfuric acid,[2)] etc. It is expected that HCDP shows three-dimensional conductivity when polymer chains are entangled at random, while quasi-one-dimensional conductivity is expected when they are well aligned each other. Actually, there are some experiments which have tried to examine the dimensionality of conduction of the tensile drawn (∼1000%) samples of HCDP films by the measurement of magnetoresistance (MR) at low temperature.[1, 2)]In these experiments the conductivities were anisotropic which were analyzed based on the formula for anisotropic three-dimensional systems. However, the resulting anisotropy turned out to be very large, which invalidate the original assumption of anisotropic three-dimensionality, i.e. the closed Fermi surface with the anisotropic mass. Instead, the results seem to indicate that the Fermi surface is open for which there have been few theoretical studies on MR.[3, 4, 5, 6, 7, 8)]

In this paper, the weak field MR for such systems with open Fermi surfaces are theoretically studied by use of the Wigner representation.

The field theoretical studies of weak-localization (WL) effects[9)] on MR have discussed by Hikami *et al.* [10)] and Kawabata[11)] for two- and three-dimensional metallic conductors, respectively. In these studies where the closed Fermi surfaces are assumed the quantum corrections to the conductivity given by the Cooperon propagators have been easily calculated even in the presence of a

magnetic field in terms of the Landau quantization. For systems with open Fermi surfaces, on the other hand, the eigenvalues of the Cooperon propagator can not be explicitly given. In order to overcome this difficulty and to study MR systematically we make use of the Wigner representation.

In §2 a brief review of the preceding theory for three-dimensional systems is given, and studies of quasi-one-dimensional systems by the Wigner representation are given in §3. The asymptotic forms of the MR in three- and one-dimensional limit and summary are given in §4 and §5, respectively.

We take a unit of $\hbar = 1$.

§2. Magnetoresistance in Three-Dimensional Systems

For three-dimensional systems, we take the model Hamiltonian,

$$\mathcal{H} = \frac{\boldsymbol{p}^2}{2m} + u \sum_l \delta(\boldsymbol{r} - \boldsymbol{R}_l), \tag{2.1}$$

where u is the strength of the short range impurity potential and $\boldsymbol{R}_l$ is the impurity site. We will consider the quantum correction term for the conductivity in the order of $(\varepsilon_{\rm F}\tau_0)^{-1}$, where $\varepsilon_{\rm F}$ is the Fermi energy and τ_0 is the relaxation time due to elastic scattering by impurities given in Fig. 1. In this figure dashed lines and a cross represent impurity potentials and the averaging procedure over the distribution of impurities. This τ_0 is given as follows,

$$\tau_0^{-1} = 2\pi n_i u^2 N(0), \tag{2.2}$$

where n_i is the density of impurities and $N(0)$ is the density of state per spin at the Fermi energy.

Fig. 1. Self-energy correction due to the impurity scattering.

The weak-localization effect can be calculated by the summation of so-called maximally crossed diagrams as given in Fig. 2. In these diagrams the ladder part (see Fig. 3) which is called the "Cooperon" represents the quantum interference effect between two electrons having nearly opposite

Fig. 2. Weak-localization correction due to the "Cooperon".

Fig. 3. The Cooperon representing the quantum interference effect.

wave number. The Cooperon is singular when $\varepsilon_n(\varepsilon_n + \omega_l) < 0$ where $\varepsilon_n = (2n+1)\pi k_{\mathrm{B}} T$, $\omega_l = 2l\pi k_{\mathrm{B}} T$ and k_{B} is Boltzmann constant, and in this case it is written as follows,

$$D_c(\boldsymbol{q}, \omega_l) = \frac{1}{2\pi N(0)\tau_0^2} \frac{1}{D\boldsymbol{q}^2 + |\omega_l| + 1/\tau_\varepsilon}, \tag{2.3}$$

where $D = 2\varepsilon_{\mathrm{F}}\tau_0/3m$ is the diffusion constant and τ_ε is the phase relaxation time due to inelastic scattering introduced phenomenologically. Then the quantum correction to the conductivity (Fig. 2) is given by

$$\frac{\Delta\sigma}{\sigma_0} = -2\ \tau_0^2\ \mathrm{Tr}\ D_c(\boldsymbol{q}, 0), \tag{2.4}$$

where $\sigma_0 = 2e^2 N(0) D$ is the Drude conductivity and Tr means quantum mechanical trace, *e.g.* $\sum_q$ in the absence of the magnetic field.

In the presence of a magnetic field, H, whose strength is not so strong, in the sense $\omega_c \equiv eH/mc \ll \tau_0^{-1}$, its effects can be treated quasiclassically, i.e. $\boldsymbol{q}$ in the Cooperon is replaced by $\boldsymbol{q} + 2e\boldsymbol{A}/c \equiv \boldsymbol{\pi}$, where $\boldsymbol{A}$ is a vector potential. Fortunately, the Cooperon depends only on $\boldsymbol{\pi}^2$, so that the trace can easily be carried out by the use of the eigenstates of Landau quantization. Hence, the quantum correction is given as follows,[11)]

$$\frac{\Delta\sigma(H)}{\sigma_0} = -\frac{1}{2\pi^3 N(0)\ell^2}$$

$$\times \sum_N \int \mathrm{d}q_z \frac{1}{\dfrac{4D}{\ell^2}\left(N+\dfrac{1}{2}\right)+Dq_z^2+1/\tau_\varepsilon}, \tag{2.5}$$

where $\ell = \sqrt{c/eH}$ is the Larmor radius. Equation (2.5) is valid for both weak and strong magnetic field limit, i.e. $\ell \gg L_\varepsilon \equiv \sqrt{D\tau_\varepsilon}$ and $\ell \ll L_\varepsilon$, as long as the conditions, $\omega_c \ll \varepsilon_{\rm F}$ and $\ell \gg \sqrt{D\tau_0}$, are satisfied. Especially for weak magnetic field, we get the following asymptotic form of the magnetoconductance, $\delta\sigma(H) \equiv \Delta\sigma(H) - \Delta\sigma(0)$,

$$\frac{\delta\sigma(H)}{\sigma_0} = \frac{1}{24\pi^2 N(0)} \frac{\sqrt{D}\tau_\varepsilon^{3/2}}{\ell^4} \ \propto \ H^2. \tag{2.6}$$

If we assume the anisotropic mass, m_i $(i = x, y, z)$, the diffusion constants are defined as $D_i = 2\varepsilon_{\rm F}\tau_0/3m_i$, and $\delta\sigma(H)$ along the symmetry axis is rewritten as

$$\frac{\delta\sigma(H)}{\sigma_0} = \frac{1}{24\pi^2 N(0)} \frac{D_1}{\sqrt{D_2}} \frac{\tau_\varepsilon^{3/2}}{\ell^4}, \tag{2.7}$$

where D_1 is the geometric mean of D_is perpendicular to the magnetic field and D_2 is the diffusion constant of the direction of magnetic field.

§3. Magnetoresistance in Quasi-One-Dimensional Systems

Now, we turn to our problem of quasi-one-dimensional systems with open Fermi surfaces. We take the model Hamiltonian,

$$\mathcal{H} = \frac{p_z^2}{2m} - \alpha(\cos p_x d + \cos p_y d) + u\sum_l \delta(\boldsymbol{r} - \boldsymbol{R}_l), \tag{3.1}$$

where z-axis is the polymer chain axis, α is the band width due to the transverse hopping of electrons among chains and d is the lattice spacing perpendicular to the chain direction. The one-particle thermal Green function is given as

$$G(\boldsymbol{k}, \mathrm{i}\varepsilon_n) = \frac{1}{\mathrm{i}\varepsilon_n - [k_z^2/2m - \alpha(\cos k_x d + \cos k_y d) - \varepsilon_{\rm F}] + \mathrm{i}\,\mathrm{sgn}(\varepsilon_n)/2\tau_0}. \tag{3.2}$$

If the Fermi energy $\varepsilon_{\rm F}$ is large enough compared to the band width in the perpendicular directions, α, which is assumed throughout this paper, and then the warping of the Fermi surface can be ignored in the integration of a single particle Green function, the relaxation time due to impurity scattering is given by

$$\tau_0^{-1} = \frac{2n_i u^2}{d^2 v_{\rm F}}, \tag{3.3}$$

where $v_{\rm F} = \sqrt{2\varepsilon_{\rm F}/m}$. On the other hand, the cosine band structure has to be properly treated in the derivation of the Cooperon as follows,

$$D_c(\boldsymbol{q}, \omega_l) = \frac{n_i u^2}{1 - n_i u^2 X(\boldsymbol{q}, \omega_l)}, \tag{3.4}$$

$$
\begin{aligned}
X(\boldsymbol{q},\omega_l) &= \int \frac{\mathrm{d}\boldsymbol{k}}{(2\pi)^3}\ G(\boldsymbol{k},\mathrm{i}\varepsilon_n+\mathrm{i}\omega_l)\ G(\boldsymbol{q}-\boldsymbol{k},\mathrm{i}\varepsilon_n) \\
&= \int \frac{\mathrm{d}\boldsymbol{k}}{(2\pi)^3} \frac{1}{\mathrm{i}(\varepsilon_n+\omega_l)-\{k_z^2/2m-\alpha(\cos k_x d+\cos k_y d)-\varepsilon_{\mathrm{F}}\}+\mathrm{i}/2\tau_0} \\
&\quad \times \frac{1}{\mathrm{i}\varepsilon_n-\{(q_z-k_z)^2/2m-\alpha\left[\cos(q_x-k_x)d+\cos(q_y-k_y)d\right]-\varepsilon_{\mathrm{F}}\}-\mathrm{i}/2\tau_0}, \qquad (3.5)
\end{aligned}
$$

where $X(\boldsymbol{q},\omega_l)$ is the polarization function. The result of the integration with respect to k_z is given as follows under the conditions, $\varepsilon_{\mathrm{F}} \gg \alpha$ and $1 \gg \alpha\tau_0|\sin\frac{q_{x,y}d}{2}|, \quad v_{\mathrm{F}}^2\tau_0^2 q_z^2, \quad |\omega_l|\tau_0$,

$$
\begin{aligned}
X(\boldsymbol{q},\omega_l) &\simeq \frac{2}{v_{\mathrm{F}}}\int \frac{\mathrm{d}k_x\mathrm{d}k_y}{(2\pi)^2} \frac{1}{\omega_l+\dfrac{1}{\tau_0}+v_{\mathrm{F}}^2\tau_0 q_z^2+2\mathrm{i}\alpha\left[\sin\dfrac{(2k_x-q_x)d}{2}\sin\dfrac{q_x d}{2}+\sin\dfrac{(2k_y-q_y)d}{2}\sin\dfrac{q_y d}{2}\right]} \\
&\simeq \frac{2\tau_0}{v_{\mathrm{F}}d^2}\left[1-2\alpha^2\tau_0^2\left(\sin^2\frac{q_x d}{2}+\sin^2\frac{q_y d}{2}\right)-v_{\mathrm{F}}^2\tau_0^2 q_z^2-|\omega_l|\tau_0\right]. \qquad (3.6)
\end{aligned}
$$

Then the Cooperon is obtained as[12, 13, 14]

$$
D_c(\boldsymbol{q},\omega_l) = \frac{d^2 v_{\mathrm{F}}}{2\tau_0^2}\frac{1}{v_{\mathrm{F}}^2\tau_0 q_z^2+\alpha^2\tau_0(2-\cos q_x d-\cos q_y d)+|\omega_l|+1/\tau_\varepsilon}. \qquad (3.7)
$$

The quantum corrections to the conductivity (Fig. 2) for each direction under the same conditions as in the derivation of the Cooperon, eq. (3.7), are as follows,

$$
\begin{aligned}
\frac{\Delta\sigma_\parallel}{\sigma_\parallel} &= -2\ \tau_0^2\ \mathrm{Tr}\ D_c(\boldsymbol{q},0), \qquad (3.8a)\\
\frac{\Delta\sigma_\perp}{\sigma_\perp} &= -2\ \tau_0^2\ \mathrm{Tr}\cos q_x d\ D_c(\boldsymbol{q},0). \qquad (3.8b)
\end{aligned}
$$

In these equations, the classical conductivities for each direction are given by

$$
\begin{aligned}
\sigma_\parallel &= 2e^2 N(0) D_\parallel, \qquad (3.9a)\\
\sigma_\perp &= 2e^2 N(0) D_\perp, \qquad (3.9b)
\end{aligned}
$$

where the symbols $\parallel$ and $\perp$ represent the directions parallel and perpendicular to the chain axis, respectively, which will be used in the following as well. Here the density of state and the diffusion constants are defined as follows,

$$
\begin{aligned}
N(0) &= \frac{1}{\pi d^2 v_{\mathrm{F}}}, \qquad (3.10a)\\
D_\parallel &= v_{\mathrm{F}}^2\tau_0, \qquad (3.10b)\\
D_\perp &= \frac{1}{2}\alpha^2 d^2\tau_0, \qquad (3.10c)
\end{aligned}
$$

which are deduced from eq. (3.7) in the continuum limit, $d \to 0$.

In the limit, $\alpha\tau_0 \ll 1$, where the warping of the Fermi surface is less than the broadening, τ_0^{-1}, (see Fig. 4 (a)), the conditions, $1 \gg \alpha\tau_0|\sin\frac{q_{x,y}d}{2}|$, are satisfied over the whole Brillouin zone, hence

Fig. 4. Fermi surfaces in the cases, $\alpha\tau_0 \ll 1$, (a), and $\alpha\tau_0 \gg 1$, (b). Here the vertical lines represent the broadening of the Fermi surface corresponding to the energy width, τ_0^{-1}.

any cutoff is not necessary in the integrations with respect to q_x and q_y in the evaluations of eq. (3.8). On the other hand, in the limit, $\alpha\tau_{0} \gg 1$, where the warping of the Fermi surface is larger than the broadening, τ_{0}^{-1}, (see Fig. 4 (b)), the conditions, $1 \gg \alpha\tau_{0}|\sin\frac{q_{x,y}d}{2}|$, required to derive eqs. (3.7) and (3.8) imply $|q_{x,y}| \lesssim (\alpha\tau_{0}d)^{-1}$. In this case, however, the main contributions to the quantum corrections, eq. (3.8), turn out to be given by the small q such as $|q| \lesssim (\alpha\sqrt{\tau_{0}\tau_{\varepsilon}}d)^{-1}$ due to the lifetime of the Cooperon, τ_ε. Since $(\alpha\sqrt{\tau_{0}\tau_{\varepsilon}}d)^{-1} < (\alpha\tau_{0}d)^{-1}$ is usually satisfied (i.e. $\tau_\varepsilon \gg \tau_{0}$), the present estimations of the quantum corrections based on eqs. (3.7) and (3.8) are justified even in this case of $\alpha\tau_{0} \gg 1$.

To obtain the MR, we replace $\boldsymbol{q}$ by $\boldsymbol{\pi} = \boldsymbol{q} + 2e\boldsymbol{A}/c$,

$$\frac{\Delta\sigma_\parallel}{\sigma_\parallel} = -\mathrm{Tr}\frac{d^2 v_{\mathrm{F}}}{D_\parallel \pi_z^2 + \alpha^2\tau_{0}(2 - \cos\pi_x d - \cos\pi_y d) + 1/\tau_\varepsilon}, \tag{3.11a}$$

$$\frac{\Delta\sigma_\perp}{\sigma_\perp} = -\mathrm{Tr}\frac{d^2 v_{\mathrm{F}}\cos\pi_x d}{D_\parallel \pi_z^2 + \alpha^2\tau_{0}(2 - \cos\pi_x d - \cos\pi_y d) + 1/\tau_\varepsilon}. \tag{3.11b}$$

Here we must be careful to treat π_is because of their noncommutability,

$$[\pi_i, \pi_j] = \mathrm{i}\frac{2}{\ell^2}\varepsilon_{ijk}, \tag{3.12}$$

where k is the direction of magnetic field and ε_{ijk} is Levi-Civita's totally antisymmetric tensor. Since π_is are contained in cosine terms in our Cooperon, we cannot use the Landau quantization method and it is impossible to study MR for arbitrary field. However for studies in weak magnetic field, the method of Wigner representation[15)] is most suited, because it is a systematic method of expanding physical quantities in terms of the small parameter which is the value of the commutator of canonical variables. Moreover as it turned out, the MR in a weak field yields important information on the degree of the alignment of the polymer and the phase relaxation time.

In the Wigner representation, the trace of some physical quantity, $A(\hat{p}, \hat{q})$, which is given as a function of canonical variables, $\hat{p}$ and $\hat{q}$ satisfying $[\hat{p}\ ,\ \hat{q}] = -\mathrm{i}c$, can be obtained by replacing quantum operators to corresponding classical differential operators operating on 1, and integrating the quantity over classical variables, p and q,

$$\mathrm{Tr}\ A(\hat{p}, \hat{q}) = \frac{1}{2\pi c} \int \mathrm{d}p \mathrm{d}q A \left(p + \frac{c}{2\mathrm{i}} \frac{\partial}{\partial q}\ ,\ q - \frac{c}{2\mathrm{i}} \frac{\partial}{\partial p} \right) \cdot 1. \tag{3.13}$$

Fig. 5. Five possible configurations of current and field with respect to the chain direction in the measurement of MR.

In our case of MR in quasi-one-dimensional systems, two components of $\hat{\pi}$ perpendicular to the magnetic field correspond to $\hat{p}$ and $\hat{q}$ in eq. (3.13). For each of the five possible configurations as shown in Fig. 5 we have to replace the operators as follows,

$$\hat{\boldsymbol{\pi}} \to \boldsymbol{\pi} + \frac{1}{\mathrm{i}\ell^2} \boldsymbol{h} \times \frac{\partial}{\partial \boldsymbol{\pi}}, \tag{3.14}$$

where $\boldsymbol{h}$ is the unit vector along the direction of magnetic field, and integrate over $\boldsymbol{\pi}$. For example the quantum correction in the config. (1) in Fig. 5, we have to evaluate the following,

$$\frac{\Delta\sigma_1}{\sigma_\parallel} = -\frac{d^2 v_{\mathrm{F}} \tau_\varepsilon}{(2\pi)^3} \int_0^\infty \mathrm{d}s \int \mathrm{d}^3\pi\ \mathrm{e}^{-s\left\{ D_\parallel \tau_\varepsilon \pi_z^2 + \alpha^2 \tau_0 \tau_\varepsilon \left[2 - \cos\left(\pi_x - \frac{1}{\mathrm{i}\ell^2} \frac{\partial}{\partial \pi_y} \right) d - \cos\left(\pi_y + \frac{1}{\mathrm{i}\ell^2} \frac{\partial}{\partial \pi_x} \right) d \right] + 1 \right\}} \cdot 1, \tag{3.15}$$

where the integrations with respect to π_x and π_y can be taken over the whole Brillouin zone.

The explicit evaluations of the quantum corrections up to the second order of H for each config-

uration result in as follows,

$$
\begin{aligned}
\frac{\Delta\sigma_1}{\sigma_\parallel} &= -\frac{1}{2\sqrt{\pi}}\sqrt{\frac{\tau_\varepsilon}{\tau_0}}\int_0^\infty \mathrm{d}s\ \mathrm{e}^{-s(4a+1)}\ [\ s^{-1/2}\ \mathrm{I}_0(2as)^2 - \frac{2}{3}\left(\frac{L_{\varepsilon\perp}^2}{\ell^2}\right)^2 s^{3/2}\ \mathrm{I}_1(2as)^2\ ],\\
\frac{\Delta\sigma_2}{\sigma_\perp} &= -\frac{1}{2\sqrt{\pi}}\sqrt{\frac{\tau_\varepsilon}{\tau_0}}\int_0^\infty \mathrm{d}s\ \mathrm{e}^{-s(4a+1)}\ [\ s^{-1/2}\ \mathrm{I}_0(2as)\ \mathrm{I}_1(2as) - \frac{2}{3}\left(\frac{L_{\varepsilon\perp}^2}{\ell^2}\right)^2 s^{3/2}\ \mathrm{I}_0(2as)\ \mathrm{I}_1(2as)\ ],\\
\frac{\Delta\sigma_3}{\sigma_\parallel} &= -\frac{1}{2\sqrt{\pi}}\sqrt{\frac{\tau_\varepsilon}{\tau_0}}\int_0^\infty \mathrm{d}s\ \mathrm{e}^{-s(4a+1)}\ [\ s^{-1/2}\ \mathrm{I}_0(2as)^2 - \frac{2}{3}\left(\frac{L_{\varepsilon\perp}L_{\varepsilon\parallel}}{\ell^2}\right)^2 s^{3/2}\ \mathrm{I}_0(2as)\ \mathrm{I}_1(2as)\ ],\\
\frac{\Delta\sigma_4}{\sigma_\perp} &= -\frac{1}{2\sqrt{\pi}}\sqrt{\frac{\tau_\varepsilon}{\tau_0}}\int_0^\infty \mathrm{d}s\ \mathrm{e}^{-s(4a+1)}\ [\ s^{-1/2}\ \mathrm{I}_0(2as)\ \mathrm{I}_1(2as) - \frac{2}{3}\left(\frac{L_{\varepsilon\perp}L_{\varepsilon\parallel}}{\ell^2}\right)^2 s^{3/2}\ \mathrm{I}_1(2as)^2\ ],\\
\frac{\Delta\sigma_5}{\sigma_\perp} &= -\frac{1}{2\sqrt{\pi}}\sqrt{\frac{\tau_\varepsilon}{\tau_0}}\int_0^\infty \mathrm{d}s\ \mathrm{e}^{-s(4a+1)}\ [\ s^{-1/2}\ \mathrm{I}_0(2as)\ \mathrm{I}_1(2as) - \frac{2}{3}\left(\frac{L_{\varepsilon\perp}L_{\varepsilon\parallel}}{\ell^2}\right)^2 s^{3/2}\ \mathrm{I}_0(2as)^2\ ],
\end{aligned}
\tag{3.16}
$$

where $\Delta\sigma_i$ is the quantum correction for the i-th configuration in Fig. 5, $\mathrm{I}_0(z)$ and $\mathrm{I}_1(z)$ are the modified Bessel functions, $L_{\varepsilon\parallel} \equiv \sqrt{D_\parallel \tau_\varepsilon}$ and $L_{\varepsilon\perp} \equiv \sqrt{D_\perp \tau_\varepsilon}$ are the phase relaxation lengths for each direction, and

$$
a \equiv \frac{1}{2}\alpha^2 \tau_0 \tau_\varepsilon = \left(\frac{L_{\varepsilon\perp}}{d}\right)^2 \tag{3.17}
$$

is the "dimensionality parameter" whose meaning is discussed below. In each of eqs. (3.16), the first term in the integral is the WL correction in the absence of the magnetic field, $\Delta\sigma_i(0)$, and the second term is the magnetoconductance, $\delta\sigma_i(H) \equiv \Delta\sigma_i(H) - \Delta\sigma_i(0)$. The expansion parameters are $L_{\varepsilon\perp}^2/\ell^2$ for $H\|z$ and $L_{\varepsilon\perp}L_{\varepsilon\parallel}/\ell^2$ for $H\perp z$, respectively. This is easily understood because the magnetic field always affect electrons through the orbital motion within the plane perpendicular to the field.

The parameter, a, represents the dimensionality in the sense of the quantum interference effects due to the Cooperon, and its physical meaning is how many chains electrons can hop through with their coherency kept. The interference of electrons is three-dimensional if a is large, $a \gg 1$, even though the Fermi surface is open because electrons can move among many chains by diffusive motion until they lose their phase memory. On the other hand, it is one-dimensional if a is small, $a \ll 1$, since electrons cannot keep coherency even in a single hopping.

§4. The Asymptotic Forms

In this section, the asymptotic forms of the conductivity in three and one dimensions are elucidated:

4.1 Three-dimensional limit

The three-dimensional limit, $a \gg 1$, of eqs. (3.16) can be obtained by using the asymptotic form of modified Bessel function, and the results are as follows,

$$
\begin{aligned}
\frac{\Delta\sigma_1}{\sigma_\parallel} &= -\frac{1}{2\pi\alpha\tau_0}\left[1.61\sqrt{\pi} - \frac{1}{\alpha\tau_0}\sqrt{\frac{\tau_0}{\tau_\varepsilon}}\right] + \frac{1}{24\pi}\sqrt{\frac{\tau_\varepsilon}{\tau_0}}\left(\frac{dL_{\varepsilon\perp}}{\ell^2}\right)^2, \\
\frac{\Delta\sigma_2}{\sigma_\perp} &= -\frac{1}{2\pi\alpha\tau_0}\left[0.41\sqrt{\pi} - \frac{1}{\alpha\tau_0}\sqrt{\frac{\tau_0}{\tau_\varepsilon}}\right] + \frac{1}{24\pi}\sqrt{\frac{\tau_\varepsilon}{\tau_0}}\left(\frac{dL_{\varepsilon\perp}}{\ell^2}\right)^2, \\
\frac{\Delta\sigma_3}{\sigma_\parallel} &= -\frac{1}{2\pi\alpha\tau_0}\left[1.61\sqrt{\pi} - \frac{1}{\alpha\tau_0}\sqrt{\frac{\tau_0}{\tau_\varepsilon}}\right] + \frac{1}{24\pi}\sqrt{\frac{\tau_\varepsilon}{\tau_0}}\left(\frac{dL_{\varepsilon\parallel}}{\ell^2}\right)^2, \qquad (4.1) \\
\frac{\Delta\sigma_4}{\sigma_\perp} &= -\frac{1}{2\pi\alpha\tau_0}\left[0.41\sqrt{\pi} - \frac{1}{\alpha\tau_0}\sqrt{\frac{\tau_0}{\tau_\varepsilon}}\right] + \frac{1}{24\pi}\sqrt{\frac{\tau_\varepsilon}{\tau_0}}\left(\frac{dL_{\varepsilon\parallel}}{\ell^2}\right)^2, \\
\frac{\Delta\sigma_5}{\sigma_\perp} &= -\frac{1}{2\pi\alpha\tau_0}\left[0.41\sqrt{\pi} - \frac{1}{\alpha\tau_0}\sqrt{\frac{\tau_0}{\tau_\varepsilon}}\right] + \frac{1}{24\pi}\sqrt{\frac{\tau_\varepsilon}{\tau_0}}\left(\frac{dL_{\varepsilon\parallel}}{\ell^2}\right)^2.
\end{aligned}
$$

These are identical with the conclusions of preceding theories of WL and weak field MR in three-dimensional systems,[9, 10, 11, 12, 13, 14] with the density of state, $N(0)$, and the anisotropic tensor components of the diffusion constants, $D_\parallel$ and $D_\perp$, as given in eq. (3.10), *e.g.* the substitution of them for eq. (2.7) gives the second terms, $\delta\sigma_i(H)$, of eqs. (4.1). This is expected because in the limit, $\alpha^2\tau_0\tau_\varepsilon \gg 1$, the main contribution to the integration of the Cooperon is given by small q such as $|q_x|, |q_y| \lesssim (\alpha\sqrt{\tau_0\tau_\varepsilon}d)^{-1}$. Therefore our formulae, *e.g.* eqs. (3.7) and (3.8), turn out to be the same as those in anisotropic three-dimensional systems shown in §2. This is the reason why the quantum corrections of the systems with $\alpha\tau_0 \gg 1$ are given by those of the anisotropic three-dimensional systems even though the Fermi surface is open, since $\alpha^2\tau_0\tau_\varepsilon \gg 1$ because of $\tau_\varepsilon \gg \tau_0$.

4.2 One-dimensional limit

When the system becomes one-dimensional, $a \ll 1$, the asymptotic forms are given as

$$\begin{aligned}
\frac{\Delta\sigma_1}{\sigma_\parallel} &= -\frac{1}{2}\sqrt{\frac{\tau_\varepsilon}{\tau_0}} + \frac{35}{16}\sqrt{\frac{\tau_\varepsilon}{\tau_0}}\, a^4 \left(\frac{d^2}{\ell^2}\right)^2, \\
\frac{\Delta\sigma_2}{\sigma_\perp} &= -\frac{1}{8}\sqrt{\frac{\tau_\varepsilon}{\tau_0}}\, a + \frac{5}{8}\sqrt{\frac{\tau_\varepsilon}{\tau_0}}\, a^3 \left(\frac{d^2}{\ell^2}\right)^2, \\
\frac{\Delta\sigma_3}{\sigma_\parallel} &= -\frac{1}{2}\sqrt{\frac{\tau_\varepsilon}{\tau_0}} + \frac{5}{8}\sqrt{\frac{\tau_\varepsilon}{\tau_0}}\, a^2 \left(\frac{dL_{\varepsilon\parallel}}{\ell^2}\right)^2, \\
\frac{\Delta\sigma_4}{\sigma_\perp} &= -\frac{1}{8}\sqrt{\frac{\tau_\varepsilon}{\tau_0}}\, a + \frac{35}{16}\sqrt{\frac{\tau_\varepsilon}{\tau_0}}\, a^3 \left(\frac{dL_{\varepsilon\parallel}}{\ell^2}\right)^2, \\
\frac{\Delta\sigma_5}{\sigma_\perp} &= -\frac{1}{8}\sqrt{\frac{\tau_\varepsilon}{\tau_0}}\, a + \frac{1}{4}\sqrt{\frac{\tau_\varepsilon}{\tau_0}}\, a \left(\frac{dL_{\varepsilon\parallel}}{\ell^2}\right)^2.
\end{aligned} \tag{4.2}$$

As is easily seen, the second term of config. (1), $\delta\sigma_1(H)$, will be reduced most rapidly as $a \to 0$, while that of config. (5), $\delta\sigma_5(H)$, will remain larger than the others.

Hence, one can infer the value of the dimensionality parameter, a, experimentally by the comparison of the anisotropy of the magnetoconductance, $\delta\sigma(H)$. For example, the ratio of $\delta\sigma_3(H)$ and $\delta\sigma_4(H)$ will give the value of a, yielding important information about the degree of the alignment of polymer.

In addition, the temperature dependence of a thus deduced gives information on that of the phase relaxation time, τ_ε.

§5. Summary

We have developed a theory of weak field MR in quasi-one-dimensional systems which have open Fermi surfaces. Even though the effects of magnetic field on electrons with such open Fermi surface are not easy to treat, the correct results in weak field regime have been determined by use of the Wigner representation. It is to be noted that this is a rare case in which the Wigner representation is applied to a explicit calculation of the quantum transport phenomena.

We have obtained the asymptotic forms of conductivities in three- and one-dimensional limit in the sense of the quantum interference effect. We have pointed out that the dimensionality parameter, a, and thus the degree of the alignment of polymers can be inferred by studying the anisotropy of the magnetoconductance, $\delta\sigma(H)$, for five possible configurations. Moreover, the temperature dependence of the phase relaxation time can be deduced from that of the dimensionality parameter.

In a more detailed comparison with the experiments, however, the existence of the mutual inter-

action effects has to be taken into account.[16] The Coulomb interaction associated with the spin Zeeman effect gives contributions to MR of the same order as the WL, but its sign is opposite and the scaling fields are different; i.e. $g\mu_{\rm B}H/k_{\rm B}T$ where g is the Landé g-factor and $\mu_{\rm B}$ is the Bohr magneton in the case of the interaction effects while $L_{\varepsilon\perp}^2/\ell^2$ for $H\|z$ and $L_{\varepsilon\perp}L_{\varepsilon\|}/\ell^2$ for $H\perp z$, respectively, in the present WL effects. Since $L_{\varepsilon\perp} < L_{\varepsilon\|}$ will be naturally satisfied, the scaling field of the WL effects for $H\|z$ should be larger than that for $H\perp z$, but the magnitude of these scaling fields (especially that in the case of $H\|z$) relative to that of interaction effects is not unique. In the case of refs. 1 and 2, the scaling field of the interaction effects comes between those two of the WL effects and the interaction effects are almost negligible for $H\perp z$, so that one can infer the dimensionality parameter, a, adequately from $\delta\sigma(H)$ of $H\perp z$.

Acknowledgments

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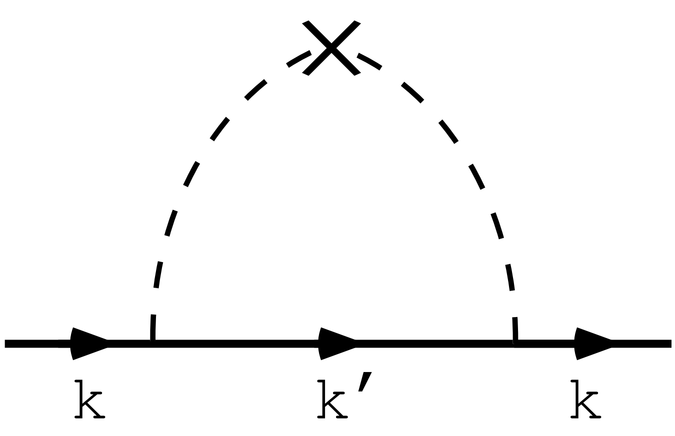
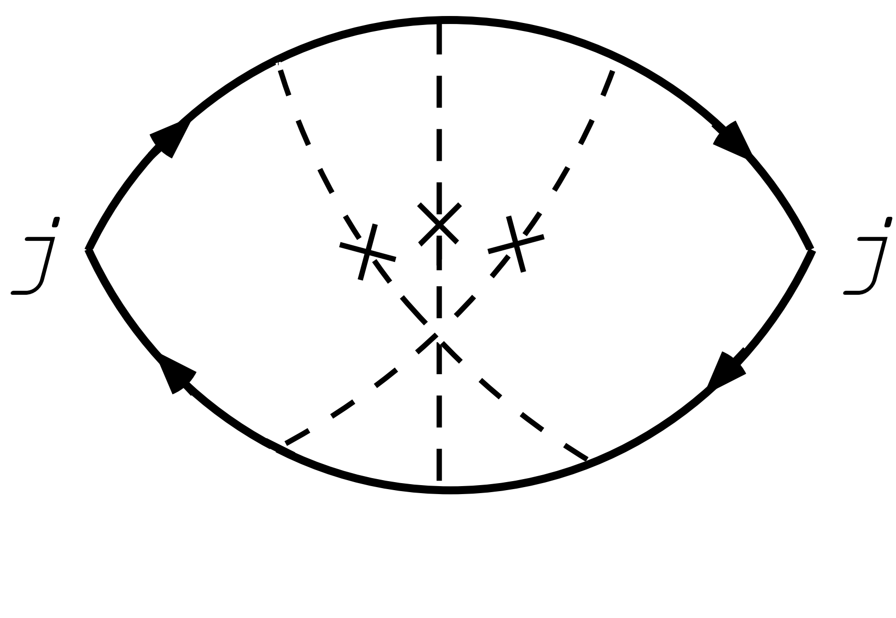
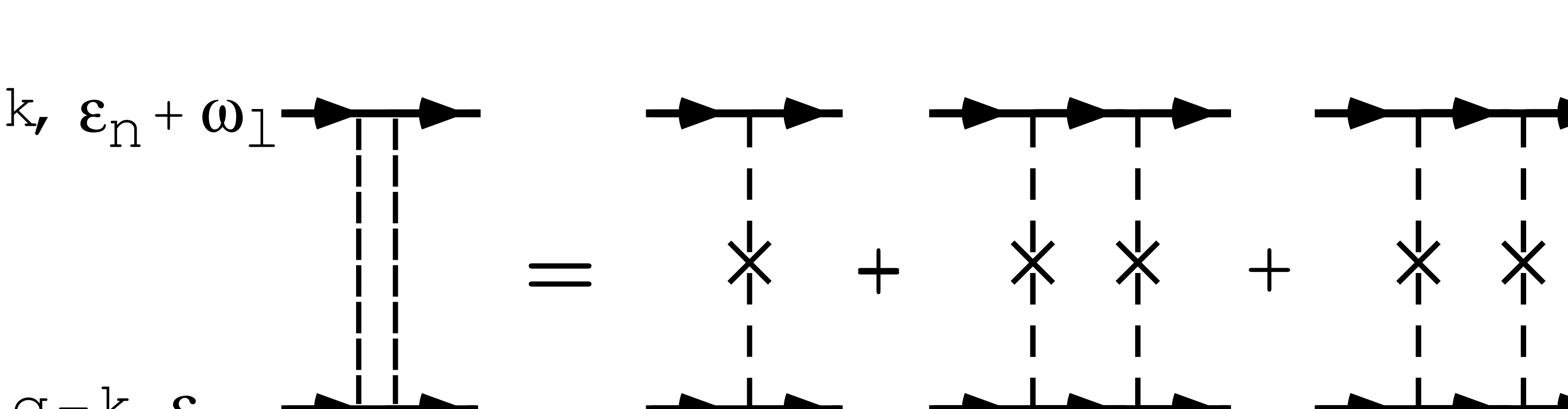
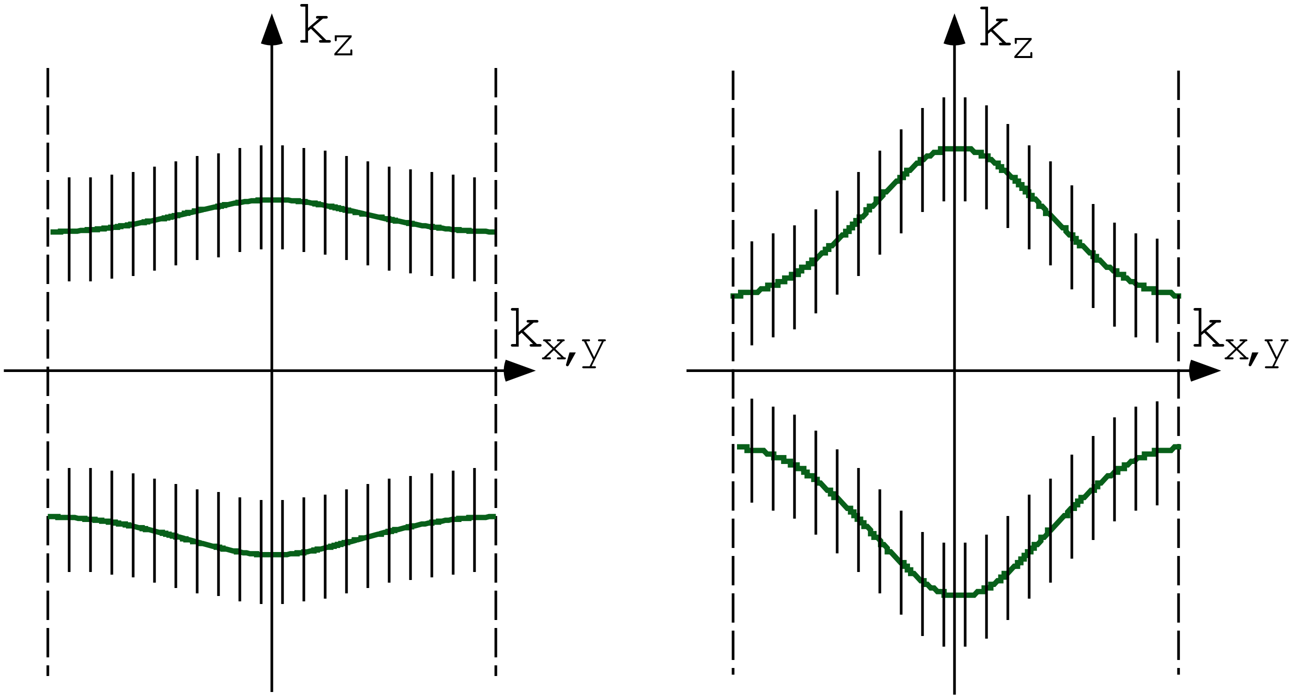
Note added—Recently, the MR in the same kind of quasi-one-dimensional systems has also been studied by C. Mauz, A. Rosch and P. Wölfle (Phys. Rev. B **56** (1997) 10953). They focused on the cases of $H\perp$chain where the Cooperon is described by Mathieu's equation and discussed various limiting cases. However, their results of one-dimensional limit are different from ours because their approximation is not justified when the coupling between chains is very weak.


k
k′
k


j
j

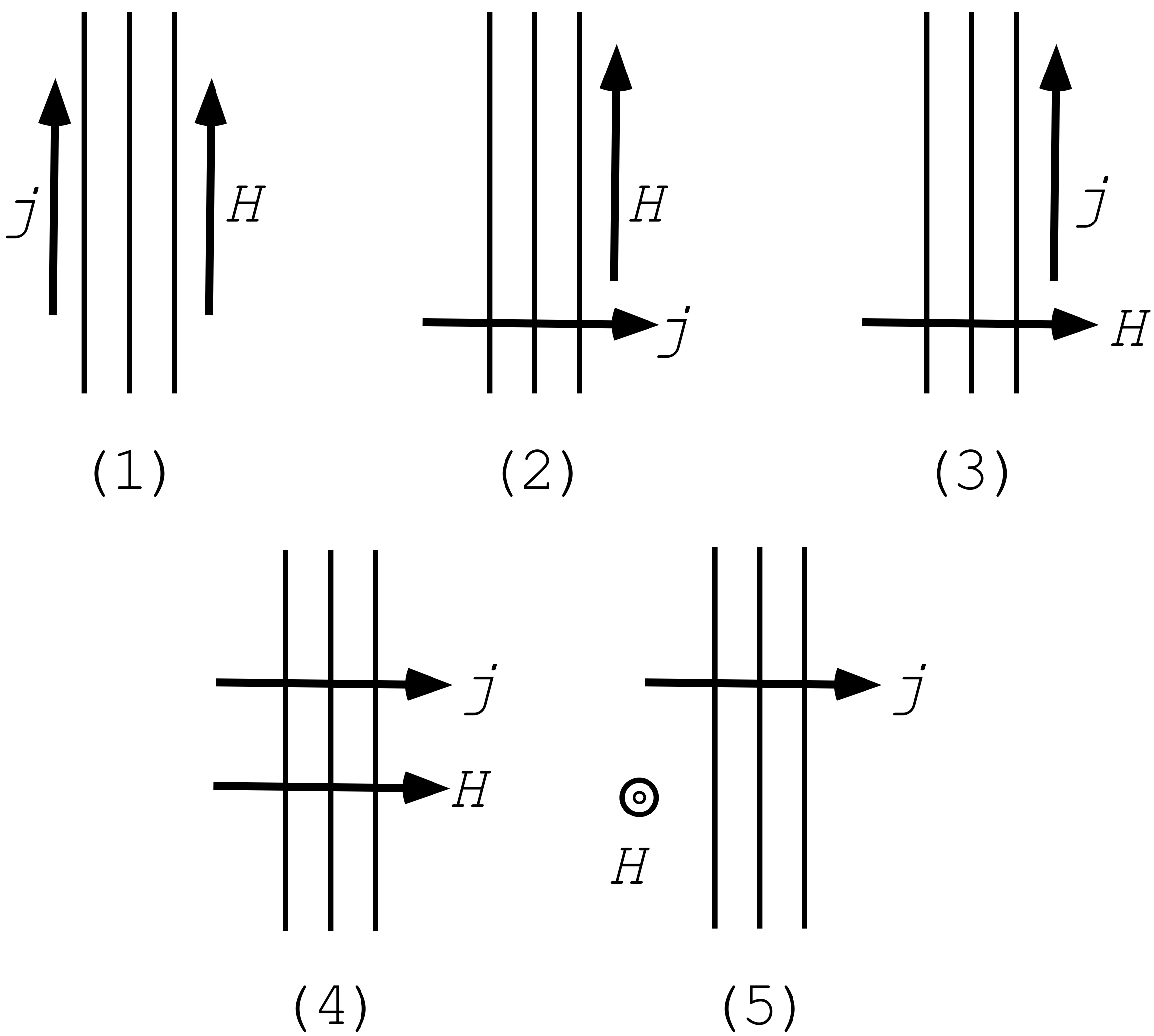


k, ε_n + ω_l
q − k, ε_n



$\alpha\,\tau_0 << 1$ $\alpha\,\tau_0 >> 1$

(a)   (b)



j
H
(1)
H
j
(2)
j
H
(3)
j
H
(4)
j
H
(5)